Quiz 3 – White Solutions

1. Three tickets are drawn at random with replacement from the box $\boxed{\boxed{1}\,\boxed{2}\,\boxed{2}\,\boxed{4}}$. Find the probability that...

(a) (3 pts) ... ***all*** 3 tickets are $\boxed{2}$s.

The probability of drawing a $\boxed{2}$ *any one time is* $\frac{1}{2}$, *and since the three draws are independent...*

$$P\left(\boxed{2}\,\boxed{2}\,\boxed{2}\right) = P\left(\boxed{2}\right)\cdot P\left(\boxed{2}\right)\cdot P\left(\boxed{2}\right) = \left(\frac{1}{2}\right)^3 = \frac{1}{8} \qquad (= 12.5\%).$$

(b) (3 pts) ... ***at least one*** of the 3 tickets is a $\boxed{4}$.

The probability of drawing at least one $\boxed{4}$ *is equal to* 1 *minus the probability of drawing* ***no*** $\boxed{4}$*s. The probability of not drawing a* $\boxed{4}$*s in one draw is* 3/4, *so similar to above, the probability of drawing no* $\boxed{4}$*s in three draws is...*

$$P\left(no\ \boxed{4}\ s\right) = \left(\frac{3}{4}\right)^3 = \frac{27}{64} \qquad (= 42.1875\%)$$

which means that the probability of drawing at least one $\boxed{4}$ *in three draws is*

$$1 - P\left(no\ \boxed{4}\ s\right) = 1 - \frac{27}{64} = \frac{37}{64} \qquad (= 57.8125\%).$$

2. (6 pts) One ticket is drawn at random from each of the two boxes

$$A \quad \boxed{\boxed{1}\,\boxed{2}\,\boxed{2}\,\boxed{4}} \qquad\qquad B \quad \boxed{\boxed{1}\,\boxed{2}\,\boxed{3}\,\boxed{4}}.$$

Find the probability that the number drawn from box A is bigger than the number drawn from box B.

There are a total of $16 = 4\cdot 4$ *possible pairs of tickets that can be drawn, with one from* A *and one from* B. *In exactly* 5 *of these* 16 *pairs is the* A *ticket bigger than the* B *ticket:*

$$(A2, B1),\ (A2, B1),\ (A4, B1),\ (A4, B2)\ \textit{ and }\ (A4, B3).$$

So the probability that the A *ticket is bigger than the* B *ticket is* $\frac{5}{16} = 31.25\%$.

3. (4 pts) A box contains 100 tickets: 60 $\boxed{1}$s and 40 $\boxed{0}$s. Tickets are drawn from the box at random with replacement, and you win a dollar if more $\boxed{0}$s are drawn than $\boxed{1}$s. There are two choices:

(i) 10 draws are made from the box.

(ii) 100 draws are made from the box.

Which choice gives a better chance of winning that dollar, (i) or (ii), or do they both give the same chance of winning?

To win a dollar, we need more $\boxed{0}$*s than* $\boxed{1}$*s, so we need the proportion of* $\boxed{0}$ *to be* ***more than*** 50%. *According to the law of averages, the more tickets are drawn, the more likely it is that the proportion of* $\boxed{0}$*s drawn will be close to the proportion of* $\boxed{0}$*s in the box — 40% in this case, which is* ***less than*** 50%. *This means that to win a dollar in this game, the fewer draws the better, so Choice (i) is better.*

4. 100 draws are made at random with replacement from the box $\boxed{\boxed{0}\,\boxed{2}\,\boxed{3}\,\boxed{4}\,\boxed{6}}$.

(a) (2 pts) Find the ***expected value*** of the sum of the draws.

The expected value for the sum of 100 *draws from this box is*

$$EV(sum) = 100 \times Avg(box) = 100 \times \frac{0+2+3+4+6}{5} = 300.$$

(b) (2 pt) What is the probability that the ***observed sum*** of the draws is between 280 and 320 — about ~~43%~~, about $\boxed{68\%}$ or about ~~95%~~? Why?

The possible sums of 100 *draws have an approximately normal distribution with average equal the the expected value and standard deviation equal to the standard error:*

$$SE(sum) = \sqrt{100} \times SD(box) = 10 \times \sqrt{\frac{(0-3)^2 + (2-3)^2 + (3-3)^2 + (4-3)^2 + (6-3)^2}{5}} = 20.$$

So the probability that the observed sum falls in the range 280 *to* 320 *is the probability of the observed sum falling within one SE of the EV (one SD of average), which is approximately* 68% *because of the approximately normal distribution of the observable sums.*